647510 **BSc6MatC602x** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
March/April-2025 (NEW COURSE)
Mathematical Analysis - 2 and Abstract Algebra-2 (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) Answer any two out of three. **(10)**

(1) Show that continuous image of connected set is connected.

(2) Show that set R of real numbers is uncountable.

(3) Prove that every bounded infinite subset of R has a limit point in R.

Que-1(B) Answer any one out of two. **(04)**

(1) Define compact set and using definition of compact set show that set of real numbers R is not compact.

(2) (i) set $A=\{x\in Z/x^3=x\}\cap N$ then check whether set A is compact and connected. Explain with reason.

(ii) Set $B = [-5,1)\cup(1,10]$ then check whether B is connected.

Que-2(A) Answer any two out of three. **(10)**

(1) Define Laplace Transform and prove that laplace transform operator L is a linear operator.

(2) State and prove First shifting theorem. Using this theorem Find $L[e^{-3x}(2\cos 5x-3\sin 5x)]$.

(3) Derive Formula of Laplace Transforms of the n^{th} derivative of f(t). Using this formula Find $L[t^n]$.

Que-2(B) Answer any one out of two. **(04)**

(1) Find : $L^{-1}[\frac{4s+5}{(s-1)^2\,(s+2)}]$.

(2) Find : $L^{-1}[\frac{s+29}{(s+4)(s^2+9)}]$.

Que-3(A) Answer any two out of three. **(10)**

(1) Prove that if $L\{f(t)\}=\bar{f}(s)$ then $L[\int_0^t f(u)du]=\frac{1}{s}\bar{f}(s)$.Using this result find $L^{-1}[\frac{4}{s^2+4s}]$.

(2) if $L\{f(t)\}=\bar{f}(s)$ and $\frac{f(t)}{t}$ has laplace transform then Prove That

$L\left\{\frac{f(t)}{t}\right\}=\int_s^\infty \bar{f}(s)ds$. Using this result Find $L[\frac{\sin t}{t}]$.

(3) Define Convolution of two functions and state convolution theorem. Using this theorem Find $L^{-1}[\frac{1}{s(s^2+4)}]$.

Que-3(B) **Answer any one out of two.** **(04)**

(1) Find : $L[\int_0^t te^{-4t} sin3t\, dt]$.

(2) Find : $L^{-1}[\frac{1}{(s-1)(s^2+1)}]$.

Que-4(A) **Answer any two out of three.** **(10)**

(1) Define kernel of Homomorphism of groups. Prove that if K is Normal subgroup of $\emptyset(G)$ then $\emptyset^{-1}(K)$ is Normal subgroup of G. where (G,*) and (G′,*′) be two groups and $\emptyset: G \to G'$ is Homomorphism.

(2) n is fixed positive integer and m is non zero element of ring (Z_n, $+_n$, x_n). Prove that integers m and n are not relatively prime iff m is zero divisor of ring Z_n.

(3) Show that integral domain D$[x]$ is not a field. Where D$[x]$ is set of all polynomials on integral domain D.

Que-4(B) **Answer any one out of two.** **(04)**

(1) Give an example of infinite integral domain and a finite integral domain which are fields.

(2) Find characteristic and zero divisors of ring (Z_6, $+_6$, x_6).

Que-5(A) **Answer any two out of three.** **(10)**

(1) Define irreducible polynomial over a field. Show that x^3+3x^2-8 is irreducible over Q$[x]$.

(2) Define Monic polynomial and gcd of two polynomials. Also State unique Factorization theorem for polynomials.

(3) Define Quaternion set. if a is any non-zero Quaternion number then find a^{-1}.

Que-5(B) **Answer any one out of two.** **(04)**

(1) Find G.C.D. d(x) of f(x),g(x) $\in$ $Z_5[x]$. where $f(x)=x^3+2x^2+3x+2$ and $g(x)=x^2+4$. Also express d(x) as a(x)f(x)+b(x)g(x).

(2) f(x)$\in$ $Z_5[x]$ Where $f(x)= x^4+3x^3+2x+4$. If g(x)= x+1 is a factor of f(x) in $Z_5[x]$ then find second factor of f(x) in $Z_5[x]$ using division algorithm theorem.

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